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SEMIPRIME RING WITH SYMMETRIC SKEW 3-REVERSE DERIVATION

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ABSTRACT

In this paper we introduce the notion of symmetric skew 3- reverse derivation of semi prime ring and we consider R be a non commutative 3-torsion free semi prime ring ,I be a non zero two sided ideal of R,α be an anti automorphism of R, and $D: R \times R \times R \rightarrow R$ be a symmetric skew 3-reverse derivation associated with the anti automorphismα , suppose that $[\tau(y), \alpha(y)] \epsilon Z$ for all $y \in I$, then $[\tau(y), \alpha(y)] = 0$ for all $y \in I$.

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1. Introduction

Bresar and Vukman[2] have introduced the notion of a reverse derivations and Samman and Alyamani[5] have studied some properties of semi prime rings with reverse derivations. AjadaFosner[1] have introduced the notation of symmetric skew 3- derivation of prime or semi prime rings and proved that under certain conditions a prime ring with a nonzero symmetric skew 3-derivation has to be commutative.The study of centralizing and commuting mappings on prime rings was initiated by the result of Posner [4] which states that the existence of a nonzero centralizing derivation on a prime ring implies that the ring has to be commutative. Vukman [6,7] investigated symmetric bi derivation on prime and semi prime rings in connection with centralizing mappings. C.JayaSubba Reddy[3] have studied prime ring with symmetric skew 3-reverse derivation. In this paper we proved that under certain conditions a semi prime ring with a nonzero symmetric skew 3-reverse derivation has to be commutative.

2. Preliminaries:

Throughout the paper,R will represent a ring with a center Z and α an anti automorphismof R. Let $n \geq 2$ be an integer. A ring R is said to be n-torsion free if for $x \in R, nx = 0$ implies $x = 0$ .For all $x, y \in R$ the symbol $[x, y]$ will denote the commutator $xy - yx$. Recall that a ring R is semi prime if $xRx = 0$,implies that $x = 0$.An additive map $d: R \to R$ is called derivation if $d(xy) = d(x)y + xd(y)$, for all $x, y \in R$. An additive map $d: R \to R$is called reverse derivation if $d(xy) = d(y)x + yd(x)$, for all $x, y \in R$ and it is called a skew derivation (α-derivation) of R associated withthe automorphism or anti automorphismα if $d(xy) = d(x)y + \alpha(x)d(y)$ for all $x, y \in R$, and it is called a skew reverse derivation (α – reverse derivation) of R associated with auto morphism or anti automorphismα if $d(xy) = xd(y) + \alpha(y)d(x)$ for all $x, y \in R$.

Before starting our main theorem, let us gives some basic definitions and well known results which we will need in our further investigation.

Let D be a symmetric 3- additive map of R, then obviously
$D(-x, y, z) = -D(x, y, z)$,for all $x, y, z \in R$.(1)
Namely, for all $y, z \in R$,
the map $D(., y, z): R \to R$ is an endomorphism of the additive group of R.
The map $\tau: R \to R$ defined by $\tau(x) = D(x, x, x)$, $x \in R$ is called trace of D.
Note that τ is not additive on R. But for all $x, y \in R$, we have
$\tau(y + x) = \tau(y) + \tau(x) + 3D(y, x, x) + 3D(x, y, y)$
Recall also that by (1),τ is odd function
More precisely,for all $x, y, z, u, v, w \in R$, we have
$D(xu, y, z) = xD(u, y, z) + \alpha(u)D(x, y, z)$,
$D(x, yv, z) = yD(x, v, z) + \alpha(v)D(x, y, z)$,
$D(x, y, zw) = zD(u, y, w) + \alpha(w)D(x, y, z)$.

Of course, if D is symmetric, then the above three relations are equivalent to each other.
To prove the main theorem, first we prove the following lemma:

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Lemma: Let R be a prime ring and $a, b \in R$. If $[y, b]a = 0$, for all $y \in R$, then either $a = 0$ or $b \in Z$.

Proof: We know that

$[xy, b]a = 0$

$x[y, b]a + [x, b]ya = 0$

$[x, b]ya = 0$ for all $x, y \in R$.

Thus $[x, b]Ra = 0, x \in R$ and since R is prime, either $a = 0$ or $b \in Z$.

3. Main Result

Theorem 1: Let R be a noncommutative 3-torsion free semiprime ring, I be a nonzero two-sided ideal of R, α be an antiautomorphismof R, and $D: R \times R \times R \to R$ be a symmetric skew 3-reverse derivation associated with the anti automorphism α. Suppose that the trace function τ is commuting on I and $[\tau(y), \alpha(y)] \epsilon Z$ for all $y \epsilon I$. Then $[\tau(y), \alpha(y)] = 0$ for all $y \epsilon I$.

Proof: Given that $[\tau(y), \alpha(y)] \epsilon Z$ for all $y \in I$.(1)

By the linearizing (1), we get

$[\tau(y + x), \propto (y + x)] \in Z$

$[(\tau(y) + \tau(x) + 3D(y, x, x) + 3D(x, y, y)), \alpha(y) + \alpha(x)] \in Z$

$[\tau(y), \alpha(x)] + [\tau(x), \alpha(y)] + [3D(y, x, x), \alpha(y)] + [3D(y, x, x), \alpha(x)] +$
$[3D(x, y, y), \alpha(y)] + [3D(x, y, y), \alpha(x)] \epsilon Z.$
(2)

Replace y by $-y$ in (2) we get

$-[\tau(y), \alpha(x)] - [\tau(x), \alpha(y)] + [3D(y, x, x), \alpha(y)] - [3D(y, x, x), \alpha(x)] -$
$[3D(x, y, y), \alpha(y)] + [3D(x, y, y), \alpha(x)] \epsilon Z.$
(3)

Now adding equation (2) & (3), we get

$[D(y, x, x), \alpha(y)] + [D(x, y, y), \alpha(x)] \epsilon Z$ for all $x, y \epsilon I$. (4)

Replacing y by $(y + x)$ in equation (4) and using (1) and (4), we get

$[D(y + x, x, x), \alpha(y + x)] + [D(x, y + x, y + x), \alpha(x)] \epsilon Z$

$[D(y, x, x), \alpha(y)] + [D(y, x, x), \alpha(x)] + [\tau(x), \alpha(y)] + [\tau(x), \alpha(x)] + [D(x, y, y), \alpha(x)]$
$+ [D(x, y, x), \alpha(x)] + [D(x, x, y), \alpha(x)] + [\tau(x), \alpha(x)] \in Z$

$[\tau(x), \alpha(y)] + 3[D(y, x, x), \alpha(x)] \epsilon Z$ for all $x, y \epsilon I$. (5)

Substituting y by yx in equation (5), we obtain

$[\tau(x), \alpha(yx)] + 3[D(yx, x, x), \alpha(x)] \in Z$

$[T(x), \alpha(x)\alpha(y)] + 3[(yD(x, x, x) + \alpha(x)D(y, x, x)), \alpha(x)] \in Z$

$[\tau(x), \alpha(x)]\alpha(y) + \alpha(x)[\tau(x), \alpha(y)] + 3[(y\tau(x) + \alpha(x)D(y, x, x)), \alpha(x)] \in Z$

$\alpha(x)([\tau(x), \alpha(y)] + 3[D(y, x, x), \alpha(x)]) + (\alpha(y) + 3y)[\tau(x), \alpha(x)] + 3[y, \alpha(x)]\tau(x) \in Z$

Therefore,

$0=[\alpha(x)([\tau(x),\alpha(y)]+3[D(y,x,x),\alpha(x)]),\alpha(x)]+[(\alpha(y)+3y)[\tau(x),\alpha(x)],\alpha(x)]$
$\quad+3[[y,\alpha(x)]\tau(x),\alpha(x)]$
$[(\alpha(y)+6y),\alpha(x)][\tau(x),\alpha(x)]+3[[y,\alpha(x)],\alpha(x)]\tau(x)=0$for all $x,y\in I$.
Replacing y by $[\tau(x),\alpha(x)]\tau(x)$, we obtain
$0=\left[\left(\alpha([\tau(x),\alpha(x)]\tau(x))+6[\tau(x),\alpha(x)]\tau(x)\right),\alpha(x)\right][\tau(x),\alpha(x)]$
$\quad+3[[[\tau(x),\alpha(x)]\tau(x),\alpha(x)],\alpha(x)]\tau(x)$
$0=([\alpha(\tau(x)).\alpha[\tau(x),\alpha(x)],\alpha(x)]+6[[\tau(x),\alpha(x)][\tau(x),\alpha(x)]])[\tau(x),\alpha(x)]$
$\quad+3[[\tau(x),\alpha(x)][\tau(x),\alpha(x)],\alpha(x)]\tau(x)+[[\tau(x),\alpha(x)],\alpha(x)]\tau(x)$
$0=[\alpha(\tau(x)),\alpha(x)]\alpha([\tau(x),\alpha(x)])[\tau(x),\alpha(x)]+6[\tau(x),\alpha(x)]^3.$

Since τ is commuting on I, we have
$2[\tau(x),\alpha(x)]^3=0$, for all $x\in I$.
It follows that

$(2[\tau(x),\alpha(x)]^2)R(2[\tau(x),\alpha(x)]^2)=0$.
Since R is semiprime, we have
$2[\tau(x),\alpha(x)]^2=0$, for all$x\in I$.(6)
On the other hand, taking $y=x^2$ in equation (5), we get
$[\tau(x),\alpha(x^2)]+3[D(x^2,x,x),\alpha(x)]\in Z.$
$[\tau(x),\alpha(x).\alpha(x)]+3[(xD(x,x,x)+\alpha(x)D(x,x,x)),\alpha(x)]\in Z.$
$2\alpha(x)[\tau(x),\alpha(x)]+3[x\tau(x)+\alpha(x)\tau(x),\alpha(x)]\in Z.$
$5\alpha(x)[\tau(x),\alpha(x)]+3x[\tau(x),\alpha(x)]+3[x,\alpha(x)]\tau(x)\in Z.$

Therefore,

$0=[\tau(x),5\alpha(x)[\tau(x),\alpha(x)]+3x[\tau(x),\alpha(x)]+3[x,\alpha(x)]\tau(x)]$
$0=5[\tau(x),\alpha(x)]^2+3[\tau(x),x][\tau(x),\alpha(x)]+3[\tau(x),[x,\alpha(x)]]\tau(x)$
$0=5[\tau(x),\alpha(x)]^2+3[\tau(x),x][\tau(x),\alpha(x)]+3[\tau(x),[x\alpha(x)-\alpha(x)x]]\tau(x)$
$0=5[\tau(x),\alpha(x)]^2+3[\tau(x),x][\tau(x),\alpha(x)]+3(x[\tau(x),\alpha(x)]-[\tau(x),\alpha(x)]x)\tau(x)$
$0=5[\tau(x),\alpha(x)]^2.$

Since τ is commuting on I, we get
$5[\tau(x),\alpha(x)]^2=0$, for all $x\in I$.(7)
Comparing equation (6) and equation (7), we get
$3[\tau(x),\alpha(x)]^2=0$,for all $x\in I$.
This implies $[\tau(x),\alpha(x)]^2=0$,for all $x\in I$.
Note that zero is the only nilpotent element in the center of a semiprime ring.
Thus $[\tau(x),\alpha(x)]=0$, for all $x\in I$. This completes the proof.

Corollary 1: Let R be a 3!- torsion free prime ring, I be a nonzero two-sided ideal of R, and α be an anti automorphism of R. Suppose that there exists a nonzero symmetric skew 3- reverse derivation $D:R\times R\times R\to R$ associated with the antiautomorphismα such that the trace function τ is commuting on I and $[\tau(y),\alpha(y)]\in Z$ for all $y\in I$.Then$R$ is commutative.

Proof: Suppose that R is not commutative. Then, according to Theorem 1, $[\tau(y), \alpha(y)] = 0$ for all $x \epsilon I$ and, by [3, Theorem 1], $D = 0,$ a contradiction.

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